



NoiseTech Microwaves Ltd.

A Primer on Noise Parameters and Noise-Parameter Measurements

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Introduction to noise parameters

RF designers often find themselves working on input power matching their circuits to a 50-Ohm signal source. The goal of this task is to develop a matching network that delivers most of the input power to the device. For that, the matching network is optimized such that the reflection coefficient presented by the network to the signal source is low (typically less than 10dB return loss) over the required frequency range.

RF designers who are working on low-noise amplifiers are also interested in minimizing the noise factors (noise figures in dBs) of their designs. Noise factor is defined by the signal to noise ratio at the input, SNR_{in} , to the signal to noise ratio at the output, SNR_{out} as, $F = \frac{SNR_{in}}{SNR_{out}}$. In other words, minimizing noise factor maximizes signal-to-noise ratio and sensitivity of their systems. Can the same input matching network do both: make input reflection coefficient low and make the noise figure low?

To answer this question, one needs to know the noise parameters [1] of their circuit before matching is applied. What are the noise parameters? Noise parameters describe the auto correlation and cross correlation between input and output noise waves (or voltages or currents) generated by the devices. The most general way of representing this is by using noise correlation matrices. However, in practice rather than using such matrices, noise parameters are used. The noise parameters give the same information but arguably in a more intuitive manner.

There are different ways the noise parameters are formulated. The minimum noise factor, F_{min} , is one such noise parameter. Sometimes it is replaced with its dB representation, the minimum noise figure, NF_{min} , and sometimes it is described with the minimum noise temperature, T_{min} , where $F_{min} = 1 + T_{min}/T_0$ and $T_0 = 290K$ is the reference temperature. $T_0 = 290K$ is not exactly a typical laboratory temperature but it is close and, when multiplied by Boltzmann's constant, produces 4×10^{-21} that is easy to work with, especially long ago when one only had a slide ruler to do the analysis with...

As previously mentioned, noise parameters are a more intuitive and complete way to describe the noise added by a circuit. F_{min} and NF_{min} indicate the absolute lowest noise factor and noise figure, respectively, a device can achieve. The information on how to achieve F_{min} comes from another noise parameter, Γ_{opt} , which is the optimum reflection coefficient of the signal-source that results in F_{min} . Γ_{opt} can also be rewritten in terms of the optimum impedance, Z_{opt} , or the optimum admittance, Y_{opt} .

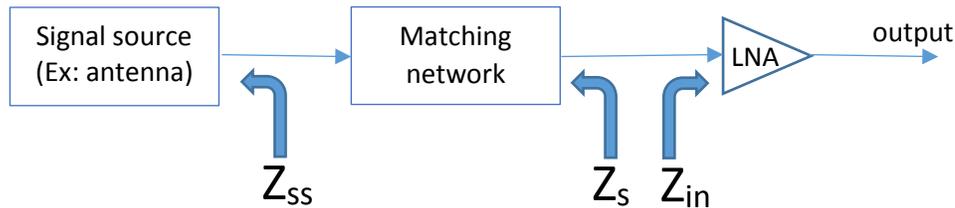


Figure 1. Block diagram of a receiver front-end.

Consider a typical front-end of a receiver shown in Figure 1, where the output impedance of the signal source is denoted as Z_{ss} . The low-noise amplifier (LNA) represents the device that is modeled with noise parameters. Z_{in} denotes LNA input impedance. Passive matching network is in general needed between the signal source and the LNA. This network has different purpose in different systems. However, most of the time is used to convert Z_{ss} to Z_s such as Z_s is complex conjugate of Z_{in} . In such a case, all output power of the signal source is delivered to the LNA. In other words, no power is reflected back to the signal source, i.e. reflection coefficient of the Matching Network and LNA is near zero.

If Z_{opt} of the LNA is known, we can determine whether the LNA noise figure (NF) comes close to NF_{min} . If it happens that $Z_s = Z_{opt}$, then NF does equal to NF_{min} and the LNA is both power matched and noise matched! If, however, this matching network creates $Z_s \neq Z_{opt}$, then the LNA noise figure will be larger than NF_{min} and the LNA is power matched but not noise matched. Alternatively, the matching network could noise match the LNA but cause a power mismatch, which results in some input power reflected back into the signal source and a reduction in power gain. When this occurs, the power gain is referred in datasheets as the “associated gain.”

How much does the noise factor increase when LNA is not noise matched? The amount of increase of the noise figure due to mismatch depends on the last noise parameter. Often, the equivalent noise resistance, R_n , is used to represent this noise parameter. With R_n , the noise factor expression of the device can be written as

$$F = F_{min} + \frac{R_n}{G_s} |Y_s - Y_{opt}|^2 \quad (1)$$

where $Y_s = 1/Z_s$, $Y_{opt} = 1/Z_{opt}$, and G_s is the real part of $Y_s = G_s + jB_s$. The noise factor can also be written as a function of reflection coefficient as

$$F = F_{min} + \frac{4R_n |\Gamma_s - \Gamma_{opt}|^2}{Z_0 |1 + \Gamma_{opt}|^2 (1 - |\Gamma_s|^2)} \quad (2)$$

where conventional conversion from impedances to reflection coefficients ($\Gamma = \frac{Z - Z_0}{Z + Z_0}$) is used in which Z_0 is the characteristic impedance, typically 50 Ω .

A designer, who is familiar with network analysis in terms of travelling waves, S-parameters, and reflection coefficients, may find the last expression of noise factor atypical. Particularly, the “+” sign, as in $|1 + \Gamma_{opt}|^2$, is not very common. Also, a designer, who wants to re-analyze the noise parameters after a short interconnecting line is added at the input, would find that adjusting the phase of Γ_{opt} based on the

transmission line length is straight forward, F_{min} does not change, but R_n does and in a not straightforward fashion.

While R_n is commonly used, it is not a fundamental noise parameter. A much better noise parameter is Lange invariant parameter N [2]–[5]. Why N is a better noise parameter? First, like F_{min} , N does not change when the device is embedded in a lossless passive network. In the example above, the short interconnect at the device input would keep both N and F_{min} unchanged. Second, N permits a check of noise parameter measurement accuracy. It has been shown in the literature that $F_{min}-1 \leq 4N$ has to be true and if noise parameters do not show this relationship, then there are errors in their measurements. Third, for many transistors $F_{min}-1 \approx 2N$. This relationship is very convenient for back-of-the-envelope analysis. Forth, N scales with transistor size in a similar way to F_{min} .

Since N , like R_n , shows the sensitivity of noise factor to mismatch between Z_s and Z_{opt} or between Y_s and complex optimum admittance $Y_{opt} = G_{opt} + jB_{opt}$, R_n and N are related. In fact, they are related via $N = R_n G_{opt}$. Then, the noise factor can be recast in terms of N as

$$F = F_{min} + \frac{4N |\Gamma_s - \Gamma_{opt}|^2}{(1 - |\Gamma_{opt}|^2)(1 - |\Gamma_s|^2)} \quad (3)$$

or

$$F = F_{min} + \frac{N}{G_{opt} G_s} |Y_s - Y_{opt}|^2 \quad (4)$$

By using $F_{min}-1 \approx 2N$, the noise factor expressions can be simplified as

$$F \approx F_{min} + \frac{2(F_{min} - 1) |\Gamma_s - \Gamma_{opt}|^2}{(1 - |\Gamma_{opt}|^2)(1 - |\Gamma_s|^2)} \quad (5)$$

or

$$F \approx F_{min} + \frac{F_{min} - 1}{2G_{opt} G_s} |Y_s - Y_{opt}|^2 \quad (5)$$

and only two noise parameters F_{min} and Y_{opt} (or Γ_{opt} or Z_{opt}).

Whether one adopts R_n or N and whether one simplifies expression for noise factor or not, the measurements of noise parameters stay the same. These can be measured by either a source pull method employing impedance tuners or impedance generators or a long-line method or a six-port method or any other methods.

Noise-parameter measurements

Noise parameters are not new. For example, IRE already had developed standard methods of measuring them in 1959 [1].

There are a few different approaches to measuring the noise parameters. Some approaches are based on representing noise signals as power waves [6]–[12], while others perform a single noise-figure measurement and fit the result to a DUT noise model that is determined analytically or experimentally using other techniques [13]–[15]. The most commonly used techniques employ source-impedance tuners that generate a few different signal-source admittances, Y_s , at the LNA input and use receivers to measure

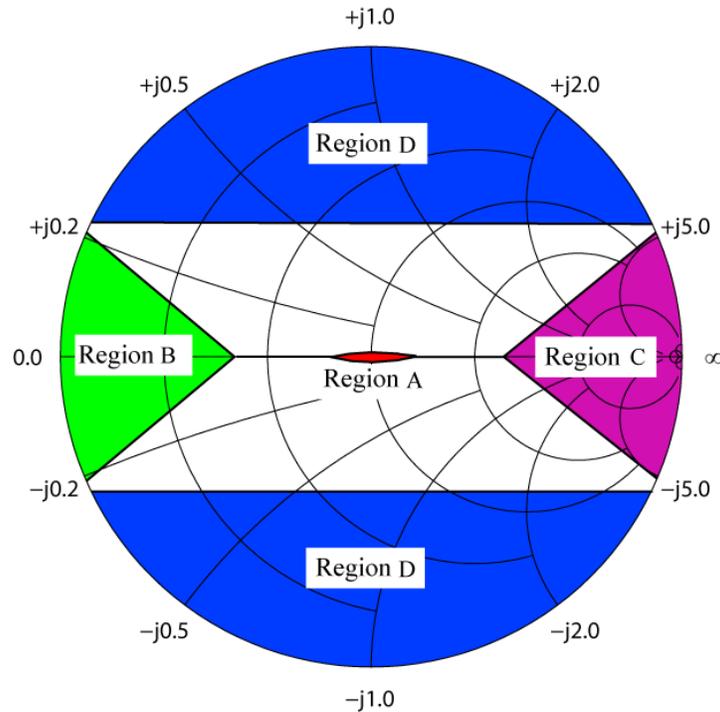


Figure 2. Regions of optimum Y_s based on [26].

the resultant noise powers at the LNA output [1],[16] – [31]. The noise parameters can be obtained using these measurements by employing data fitting techniques as described in [32]–[35].

While there are seemingly many ways of determining noise parameters, most of them are based on manipulation of $Y_s = 1/Z_s$ by varying the matching network, such as in Figure 1. By manipulating this network to generate at least four different Y_s (Y_{s1} , Y_{s2} , Y_{s3} , and Y_{s4}) and measuring noise factors (F_1 , F_2 , F_3 , and F_4) for each Y_s , equation (1) results in a system of four equations, represented as

$$\mathbf{A}\mathbf{x} = \mathbf{F} \quad (6)$$

where, for $n \geq 4$ measured complex admittances Y_s ($Y_s = G_s + jB_s$), \mathbf{A} is a $n \times 4$ matrix calculated from Y_s , \mathbf{x} is the vector of unknown noise parameters, and \mathbf{F} is a vector of measured noise factors. Noise parameters are then obtained by solving the system (6) to find the four unknowns.

As an example, for four signal-source admittances Y_{s1} , Y_{s2} , Y_{s3} , and Y_{s4} , (6) becomes

$$\begin{bmatrix} 1 & G_{s1} + \frac{B_{s1}^2}{G_{s1}} & \frac{1}{G_{s1}} & \frac{B_{s1}}{G_{s1}} \\ 1 & G_{s2} + \frac{B_{s2}^2}{G_{s2}} & \frac{1}{G_{s2}} & \frac{B_{s2}}{G_{s2}} \\ 1 & G_{s3} + \frac{B_{s3}^2}{G_{s3}} & \frac{1}{G_{s3}} & \frac{B_{s3}}{G_{s3}} \\ 1 & G_{s4} + \frac{B_{s4}^2}{G_{s4}} & \frac{1}{G_{s4}} & \frac{B_{s4}}{G_{s4}} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \quad (7)$$

where $\mathbf{F} = [F_1 \ F_2 \ F_3 \ F_4]^T$ is the array of measured noise factors for each of $Y_s = G_s + jB_s$, and A, B, C, and D are unknowns that related to the noise parameters by [26],[30]

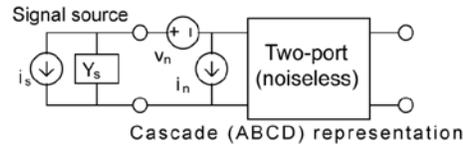


Figure 3. ABCD noise model of a device.

$$\left\{ \begin{array}{l} F_{min} = A + \sqrt{4BC - D^2} \\ R_n = B \\ G_{opt} = \frac{\sqrt{4BC - D^2}}{2B} \\ B_{opt} = -\frac{D}{2B} \\ Y_{opt} = G_{opt} + jB_{opt} \end{array} \right. \quad (8)$$

While the process appears straightforward, the practical implementation of it requires careful measurements. This is because these measurements rely on very accurate measurements of noise powers that are very low and easily contaminated by interference and measurement errors. In addition, Y_{opt} is a complex quantity and so vector calibrations are required thereby making vector-network analyzers indispensable.

Current research has been focusing on determining the optimum values of signal-source admittances Y_s such that solution of the system of equations were possible while avoiding spending significant amount of time on measurements. All previous research agreed that one of the admittances Y_s should be located near the center of the Smith Chart.

For other admittance Y_s , [18], [31] have observed that a uniformly distributed pattern of Y_s on the Smith chart resulted in reasonable noise parameter extraction with a few Y_s . Farther, [28] showed that expanding the admittance Y_s pattern in [18] to a nonuniform coverage of the Smith Chart may be beneficial. However, [28] required additional admittances Y_s to complete the measurements.

In addition, [31] results show that indeed just four admittances Y_s is sufficient to solve the system of equations and extract noise parameters. In its identification of the Y_s pattern, [31] investigated the system of equations in (6) and focused on selecting admittances Y_s that created the largest normalized determinant of \mathbf{A} in (6). While, the normalization of the determinant was not based on any physical reasons, the results in [[31], Fig. 4] showed measurement errors were reduced for some combinations of admittances and suggested that separating admittances Y_s 120deg results in a useful Y_s pattern for noise parameter measurement.

Unlike many previous methods, which sometimes employ over 20 such admittances, recent work in [26] also showed that indeed four admittances Y_s is sufficient to extract noise parameters provided that these Y_s are well selected. This work used linear algebra to determine which admittances guaranteed both a diagonally dominant \mathbf{A} in (7) and a solution to (7). Because the locations of admittances Y_s were not restricted, the optimum locations of Y_s appeared as four regions A, B, C, and D on the Smith Chart as shown in Figure 2. Since noise parameters are related to the ABCD representation of the device, noise parameters relate to the input-referred noise voltage, v_n , and noise current, i_n , as shown in Figure 3. This

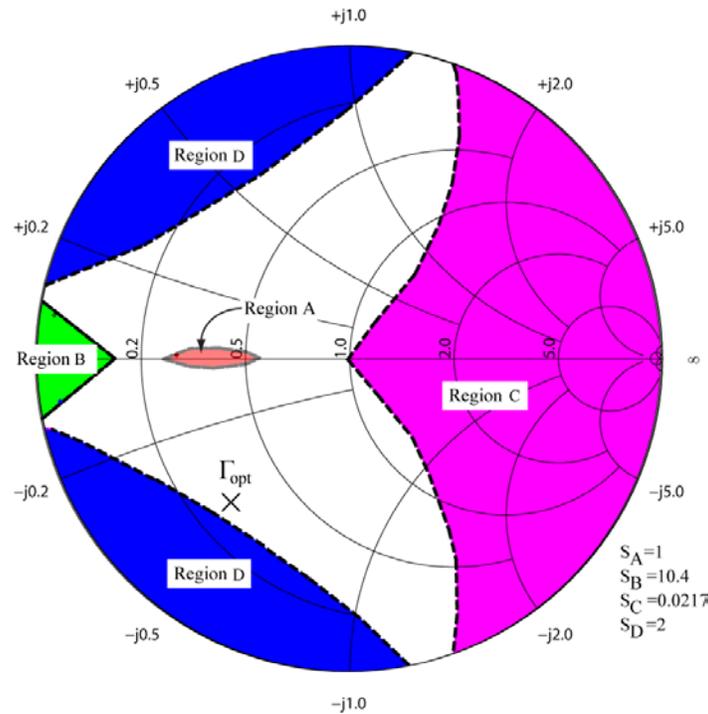


Figure 4. Example of modified regions of optimum Y_s based on [26].

relationship can be seen through the noise-correlation matrix [36], which in terms of noise parameters is written as

$$C_{ABCD} = 2kT \begin{bmatrix} R_n & \frac{F_{min} - 1}{2} - R_n Y_{s,opt}^* \\ \frac{F_{min} - 1}{2} - R_n Y_{s,opt} & R_n |Y_{s,opt}|^2 \end{bmatrix} \quad (9)$$

and in terms of v_n and i_n , it is written as

$$C_{ABCD} = \frac{1}{2\Delta f} \begin{bmatrix} v_n v_n^* & v_n i_n^* \\ v_n^* i_n & i_n i_n^* \end{bmatrix} \quad (10)$$

Coming back Figure 2, it is then concluded that the region A can be thought to correspond to a well-matched region responsible for identifying the overall noise level of the device, the region B is responsible for isolating input-referred noise voltage, v_n , component, the region C is responsible for isolating input-referred noise current, i_n , component, and the region D is responsible for determining the correlation between v_n and i_n . Farther, these regions can be modified by selecting different sets of scaling factors to increase frequency coverage of the matching network in Figure 1. One of many possible modifications to the regions is shown in Figure 4.

By demonstrating that optimum admittances Y_s for noise-parameter measurements do not have to form a constellation of single points on the Smith Chart and that the optimum regions can be numerically modified to suit certain frequency limitations of the matching network, [26] opened up the possibility of developing a truly broad band approach to measuring noise parameters. In the past, the matching networks were implemented with mechanical impedance tuners that relied on long transmission lines to provide the needed separation of Y_s . Such approach was very frequency dependent. Newly introduced

impedance generators are designed to maintain Y_s within optimum regions over large frequency ranges thereby allowing for concurrent measurement of noise parameters over very wide bands with only four impedance generator states.

Noise parameters vs noise figure

For any device, such as an LNA, noise figure is a measure of signal-to-noise ratio degradation due to the device. Noise figure depends on the output impedance Z_s of whatever circuit is driving the device, such as the matching network in Figure 1. To know how much noise figure changes when the driving port impedance Z_s changes (e.g. due to the change of antenna impedance due to objects located near the antenna), one needs to know the device noise parameters. Noise parameters give a complete picture of noise behavior of the device. They can be used to determine noise figure, but they can also be used to determine the best possible noise figure achievable with the LNA and the optimum matching to achieve it.

A low-noise-figure design of an LNA is not possible without the knowledge of transistor noise parameters. This is because without them, a designer does not have a way of calculating the LNA noise figures. Engineers can do their best work when they have the best tools and best circuit models. Noise parameters provide the best noise model, and all industry-standard simulations tools are designed to readily use them.

In summary, noise parameters:

- Provide complete picture of the best noise figure possible with a given device (typically a low-noise amplifier (LNA))
- Give designers all necessary information on how to design a matching network to achieve the lowest noise figure
- Provide ways of comparing devices in terms of their noise, ease of matching, and sensitivity to mismatch
- Predict how much noise figure penalty to expect due to mismatches at the LNA input (for example due to antennas not being exactly 50Ω)
- Allow proper calculation of noise figure for a cascade of system components
- Permit evaluation of the quality of input matching network and its ability to achieve the best noise figure.
- Ease the trade-off between power consumption and noise figure, i.e. rather than sourcing a higher power device to reduce noise figure, a better matching network may result in adequate noise figure with lower power device.

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